BAYESIAN GALAXY SHAPE ESTIMATION

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The accurate measurement of galaxy ellipticities is vital for weak lensing studies, in particular cosmic shear. We describe a Bayesian approach to this problem in which galaxies are parameterised as sums of Gaussians, convolved with a psf which is also a sum of Gaussians, following Kuijken (1999). We calculate the uncertainties in the output parameters using a Markov Chain Monte Carlo approach. We show that for a simple simulation, the ellipticity estimates do not give a biased result when averaged by statistical weight. It is shown that the uncertainties in the ellipticities are not increased by allowing freedom in the galaxy radial profile, or by allowing the photon shot noise level to be a free parameter. Finally we confirm the result of Kuijken (1999) that on changing the ellipticity of two point spread function Gaussian components, the reconstructed galaxy ellipticity is unbiased.

1 Introduction

The measurement of galaxy shapes from images is central to the field of weak gravitational lensing, in which the observed ellipticities of distant galaxies are used to infer properties of the intervening matter distribution. The circularising effect of the atmosphere must be accounted for in ground based images, as well as distortions due to telescope optics, which together are called the 'point spread function' (PSF). In the newly developing field of 'cosmic shear', where the signal is extremely small and the results are being used to constrain cosmological parameters, it is particularly important to ensure that there are negligible systematic errors. The cosmic shear signal induces ellipticities of the order of 0.01, while ground based telescopes typically induce ellipticities ~ 0.05 thus accurate correction is crucial.

Galaxy shape estimation methods were first investigated by Bonnet & Mellier (1995). Today the most widely used method is that of Kaiser, Squires and Broadhurst (1995, KSB) in which the observed weighted quadrupole moments of a galaxy are modified according to the properties of stars in the image. Improvements on the KSB method have been made by Luppino & Kaiser (1997); Hoekstra et al. (1999); and Rhodes et al. (2000). Variants of the KSB method were scrutinised in the context of cosmic shear in Bacon et al. (2001) and Erben et al. (2001) who conclude that the cosmic shear results presented thus far can be relied on, but that there are also significant limitations. For example, Bacon et al. (2001) find that the recovered shear is consistently 0.85 times the input shear and that there is a residual anti-correlation between PSF ellipticity and the corrected ellipticities of faint galaxies. Erben et al. (2001) highlight that the KSB method also breaks down for large shears ($\gamma > 0.3$).

Kaiser (2000) presented a more general approach in which more galaxies can be included in the analysis, weighted by the amount of information contained. Kuijken (1998, hereafter K98) presented a Bayesian method in which the galaxy and PSF is modelled as a sum of Gaussians and the image pixel intensities are predicted and compared to those observed. Ratnatunga, Griffiths & Ostrander (1999) have also used maximum likelihood fitting methods. Refregier and Bacon (2001) and Bernstein et al (2001) parameterise the galaxy and PSF as a sum of 'shapelets' which are linearly related to the gravitational shear. The approach in this paper follows that of K98, which is attractive because of its simple and intuitive galaxy parameterisation. We attempt to be fully Bayesian, paying particular attention to the computation of uncertainties on the measured ellipticities, which can be used as statistical weights.

The method used to extract galaxy shape parameters from a known PSF is described in Section 2. Section 3 details the results of simulations of a single galaxy, including two tests for biases. We conclude in Section 4.

2 MCMC Gaussian parameter estimation

From the probability distribution function of the model parameter values given the data we estimate the most probable parameter values with uncertainties.

2.1 Galaxy parameterisation

As in K98, galaxies are considered to be made up of sums of Gaussians. Thus the intensity as a function of position $\mathbf{x} = (x, y)$ for a galaxy is

$$B(\mathbf{x}) = \sum_i \frac{A_i}{2\pi|C_i|} \, e^{-(\mathbf{x}-\mathbf{x}_i)^{\mathrm{T}} C_i (\mathbf{x}-\mathbf{x}_i)/2} \tag{1}$$

where the covariance matrix C can be written in terms of conventional ellipse parameters as

$$C_{1,1} = 2\left(\frac{\cos^2(\theta)}{a^2} + \frac{\sin^2(\theta)}{b^2}\right) \tag{2}$$

$$C_{1,2} = \left(\frac{1}{b^2} - \frac{1}{a^2}\right) \sin(2\theta) \tag{3}$$

$$C_{2,2} = 2\left(\frac{\cos^2(\theta)}{b^2} + \frac{\sin^2(\theta)}{a^2}\right) \tag{4}$$

where the covariance matrix is symmetric. In this paper we define ellipticity to be

$$e = \frac{a-b}{a+b} \tag{5}$$

and thus the elongation parallel to the x and y axes, e_+, and the elongation along the diagonals, $e_\times$, are given by

$$e_+ = e\ \cos(2\theta) \tag{6}$$
$$e_\times = e\ \sin(2\theta) \tag{7}$$

Thus each Gaussian galaxy component has 6 parameters, which in this paper we consider to be the x, y position of the centre; e, θ, ab and the amplitude A (see Eq. 1).

2.2 Point spread function convolution

As discussed in K98, because the galaxy is a sum of Gaussians, convolution with another sum of Gaussians is analytically simple. If the PSF is written as in Eq. 1, with m components, then the convolved galaxy may also be written as in Eq. 1 but with $n \times m$ components, the new parameters of which are

$$A_{ij} = A_i\ Aj \tag{8}$$
$$C_{ij} = C_j\,(C_j + C_i)^{-1}\,C_i \tag{9}$$
$$= \frac{1}{|C_j + C_i|}\,(|C_j|\,C_i + |C_i|\,C_j) \tag{10}$$
$$\mathbf{x}_{ij} = \mathbf{x}_i + \mathbf{x}_j\ . \tag{11}$$

Thus the resulting intensity as a function of position is

$$B_{\mathrm{gal*psf}}(\mathbf{x}) = \sum_{ij} \frac{A_{ij}}{2\pi|C_{ij}|}\ e^{-(\mathbf{x}-\mathbf{x}_{ij})^{\mathrm{T}} C_{ij} (\mathbf{x}-\mathbf{x}_{ij})/2}. \tag{12}$$

2.3 Probability estimation

Consider a small section of the image, $\mathbf{D}$, containing one galaxy, for example 16×16 pixels across. Assuming the noise on each pixel is independent and

Gaussian the probability of the image given the galaxy parameters is

$$\Pr(\mathbf{D}|\mathbf{g}, \mathrm{PSF}) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\chi^2/2} \tag{13}$$

$$\chi^2 = \sum_{k,l} \frac{\left(B_{\mathrm{gal*psf}}(\mathbf{x}_{k,l}) + b - D_{k,l}\right)^2}{\sigma^2} \tag{14}$$

where σ is the noise level on the image and b is the background level, which are both treated as free parameters. (Note that is would be simple to switch to eg. Poisson noise). Thus we consider the model parameter vector $\mathbf{g}$ to consist of $\mathbf{g} = (\sigma, b, x_1, y_1, e_1, \theta_1, ab_1, A_1, \ldots, x_n, y_n, e_n, \theta_n, ab_n, A_n)$, where n is the number of Gaussian components that make up the galaxy.

Each parameter in $\mathbf{g}$ is assigned a prior which allows the conversion to the posterior probability $\Pr(\mathbf{g}|\mathbf{D}, \mathrm{PSF})$, where for simplicity we always assume the PSF is known exactly. At some additional computational cost it would be possible to generalise to the case where the PSF is known with some level of uncertainty.

Some simulated data is shown in Fig. 1. The galaxy (Fig. 1a) is made up of just one Gaussian, with $e_+ = 0.2$ and $e_\times = 0.1$. The PSF (Fig. 1b) is central, circular ($e = 0$), roughly the same size as the galaxy and is normalised to conserve galaxy flux. The resulting convolution is shown in Fig. 1c. A flat background level $b = 100$ and random Gaussian noise of standard deviation $\sigma = 20$ is added to produce the simulated data shown in Fig. 1d.

2.4 Markov-Chain Monte Carlo sampling

Markov-Chain Monte Carlo (MCMC) sampling is a method for estimating model parameters given a posterior probability distribution function. We use the BayeSys code of Maximum Entropy Data Consultants Ltd. (Skilling & Gull). The output of this code is a list of samples from the posterior probability distribution, $\mathbf{g}_i$. We do not discuss how MCMC achieves this result here, an excellent introduction to sampling methods is given in MacKay (2001). The probability distribution can be visualised by plotting points at each sample position. An example in the x, y plane is shown for a single Gaussian fit to the simulated data in Fig. 2. The number density of samples is proportional to the probability density. This figure shows that the two parameters are relatively uncorrelated, which is also found for the other parameter pairings. Perhaps not surprisingly, there is a slight degeneracy between ab and A, in which a wider, lower peaked Gaussian is degenerate with one with a narrower, higher peak.

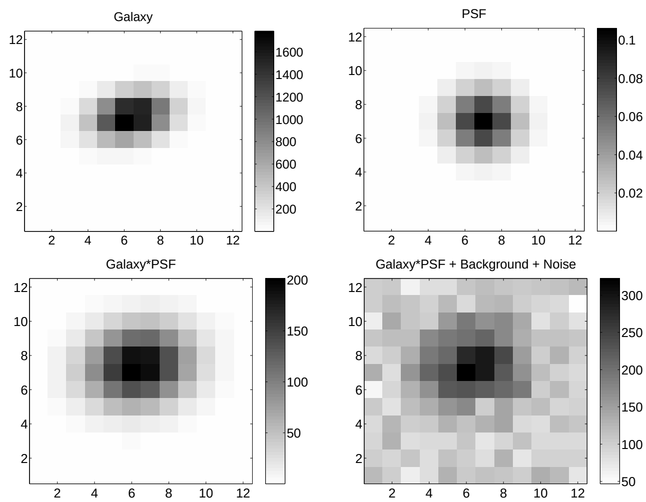

Figure 1: Simulated data.

The most probable parameter values and 68 per cent uncertainties are estimated by calculating the mean and standard deviation of the samples for each parameter. This is only strictly correct in the case that the marginal distributions are Gaussian. Otherwise this is a good indication of the uncertainty. Note that these uncertainties take into account the errors on all parameters, including the unknown x, y position of the galaxy, effectively marginalising over them.

Note also that most probable values and uncertainties can be trivially calculated for any galaxy parameters, not just the parameterisation used for the fitting. For example, e_+ for each e and θ (Eq. 7) is found by calculating e_+ for each sample, and taking the mean and standard deviation of the result, over the samples (Fig. 2 top right). Since e_+ and $e_\times$ are the quantities most closely related to the gravitational shear we focus on these two parameters in the rest of the paper.

An additional feature of this MCMC routine is that a quantity called the Evidence ($\Pr(\mathbf{D}) = \int_{\mathbf{g}} \Pr(\mathbf{D}|\mathbf{g}) \Pr(\mathbf{g}) \mathrm{d}\mathbf{g}$) is calculated during a random sampling phase of the iteration. This quantity effectively balances the goodness of fit of a model with the number of degrees of freedom, such that different parameterisations such as the number of Gaussian components may be logically decided upon.

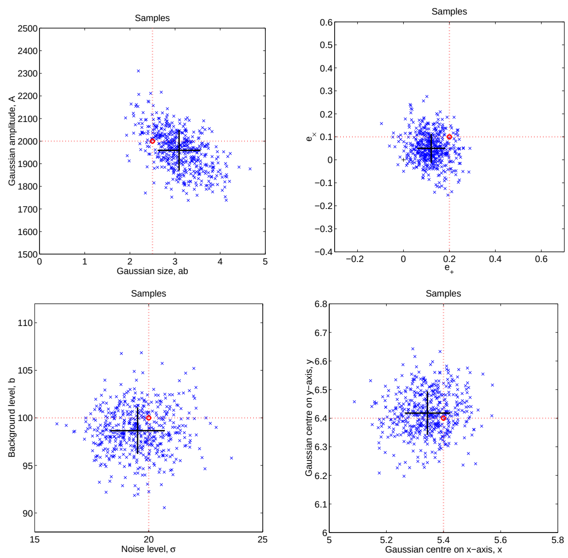

Figure 2: Samples from the posterior probability distribution.

3 Simulations

First we consider the estimation of parameters from simulations in which the model parameters are a single Gaussians for each of the PSF and the galaxy.

3.1 Single Gaussian component

We use the simulation plotted in Fig. 1. The exact PSF used in the simulation is also used as an input to the galaxy parameter estimation. The parameters fitted are $\mathbf{g} = (\sigma, b, x_1, y_1, e_1, \theta_1, ab_1, A_1)$, making 8 parameters in total. For this noise realisation we find $e_+ = 0.22 \pm 0.10$ and $e_\times = 0.21 \pm 0.10$, compared to the true values of $e_+ = 0.20$ and $e_\times = 0.10$. An important aspect of this method is that the sucess of the parameter estimation can be gauged by subtracting the reconstruction from the input data to make a residuals map. Statistics on this residuals map can be used to flag bad fits, eg. in the case where two galaxies are very close, a single galaxy is not a good fit.

To assess whether these estimates with error bars are accurate approximations to the full probability distribution we perform 100 noise realisations of the same simulation, and calculate the weighted mean values of e_+, $e_\times$ and the weighted uncertainty in the mean. After 100 noise realisations we find

$e_+ = 0.195 \pm 0.011$ and $e_\times = 0.093 \pm 0.012$, there is therefore no evidence for a problem.

3.2 Too much freedom?

For ground based images the point spread function is adequately parameterised by one or two Gaussians (a typical Moffat profile is well approximated by the sum of two Gaussians to better than one per cent). We verify that, as discussed in K98, sums of two or three Gaussian components are adequate for exponential and de Vaucouleurs galaxy profiles, given typical galaxy sizes, CCD pixelations and PSFs. In practice we tie together the centre, ellipticities and position angles of the Gaussians, although this is not required by the code. We allow complete freedom in the sizes (ab_i) and amplitudes (A_i) for each component, which has the advantage over the method of Griffiths et al. that no classification of galaxies into different types is required.

We investigate whether leaving this freedom has a significant effect on the uncertainties on the measured e_+ and $e_\times$ values. A single Gaussian was used to simulate a galaxy, and then it was reconstructed using (i) a single Gaussian and (ii) a sum of three Gaussians. Fig. 3 shows the uncertainties on e_+ and $e_\times$ as a function of galaxy luminosity for a single noise realisation. The crosses are for the single Gaussian fit and the circles for the three Gaussian fit. Clearly there is very little difference. This is perhaps not surprising, given that we might expect the errors on the radial profile to be fairly uncorrelated with the errors on the ellipticity.

In contrast to K98 we keep the noise level σ as a free parameter, and estimate it from each image. The noise level is important for estimating uncertainties in eg. e_+ and $e_\times$, and so is not relevant in K98 where the uncertainties are not considered. The number of pixels of the image used for the fitting is likely to affect how accurately the noise level can be estimated - ie. for a very large image we expect that the noise level can be determined much better than for a small image. Therefore we plot in Fig. 3 the results with and without the noise as a free parameter for a range of different simulated image sizes. For images greater than 8x8, the uncertainties on e_+ and $e_\times$ are not significantly affected by allowing the noise level to be a free parameter.

3.3 Radially varying PSF ellipticity

In the above simulations, the PSF was circular. In this subsection we verify the results of K98, who investigate the effect of a two Gaussian component PSF in which each component has a different ellipticity. K98 find significant residuals using the KSB method and much smaller residuals using their method.

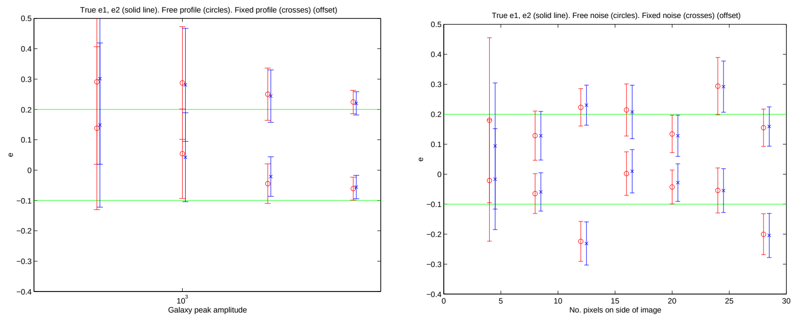

Figure 3: Left: Reconstructed ellipticity using a 3 Gaussian fit (circles), compared to the result using a 1 Gaussian fit (crosses) for a single noise realisation. The simulation contains only a single Gaussian. Points are offset for clarity. The error bars are virtually unchanged on allowing more freedom in the profile. Right: Reconstructed ellipticity as a function of image size. The circles are the results when the photon noise level is a free parameter, and the crosses are obtained when the noise level is fixed at the true value (offset for clarity).

Here we simulate a circular galaxy and convolve it with a two component PSF. One component is much larger than the other, and they have opposite values of e_+ (always $e_\times = 0$). The e_+ value is varied from zero to 0.5. We obtain the results shown in Fig. 4 which show that again we also have very small residuals. In fact these residuals are entirely accounted for by the errors due to the finite number of noise realisations performed. For comparison, in a very similar simulation, K98 found that KSB gave residuals of ~ 0.04 for a PSF ellipticity of ~ 0.5.

4 Conclusions

We have presented a conceptually simple method for estimating galaxy ellipticities, with carefully calculated uncertainty estimates. This method is intrinsically designed to cope with high noise levels on the input data and naturally allows bad fits to be identified from the residuals map.

We have assumed that galaxies have an elliptical profile, which is not observed to be the case in observations of sufficient resolution. This could be addressed if the approach were extended to other parameterisations of galaxies. Most simply the galaxy could be represented by more Gaussians, each with different centres. Alternatively the parameterisation of Refregier & Bacon (2001) and Bernstien et al. (2001) could be used.

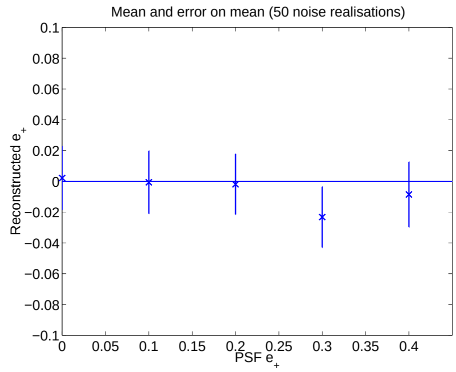

Figure 4:

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