

e.g. singular isothermal sphere

Def: $\rho(r) = \frac{\rho_0}{r^2}$ (Schneider eqn 50)
 see: (54)

$$\Sigma(\xi) = \int_{-\infty}^{\infty} dr_z \rho(\sqrt{\xi^2 + r_z^2})$$

$$= \rho_0 \int_{-\infty}^{\infty} \frac{dr_z}{(\xi^2 + r_z^2)} \quad (55)$$

$$= \frac{\rho_0}{\xi} \arctan\left(\frac{r_z}{\xi}\right) + c$$

http://www.sosmath.com/tables/ →
 integral/integ41/integ41.html (56)

$$\int_0^{\infty} \frac{dx}{x^2+a^2} = \frac{\pi}{2a} \rightarrow \text{times by 2 for } -\infty \text{ to } \infty$$

$$= \frac{\pi \rho_0}{\xi} \quad (57)$$

$$\rightarrow K(\theta) = \frac{\Sigma(D_d \theta)}{\Sigma_{crit} D_d} = \frac{\pi \rho_0}{\Sigma_{crit} D_d} \frac{1}{\theta} \quad (58)$$

$$\rightarrow (53) \rightarrow \alpha(\theta) = \frac{2}{\theta} \int_{\theta'=0}^{\theta} \frac{\pi \rho_0}{\Sigma_{crit} D_d} \frac{1}{\theta'} \theta' d\theta'$$

$$= \frac{2\pi \rho_0}{\Sigma_{crit} D_d} \frac{1}{\theta} \left[\theta' \right]_{\theta'=0}^{\theta}$$

$$= \frac{2\pi \rho_0}{\Sigma_{crit} D_d} \quad \text{i.e. const bend angle} \quad (59)$$