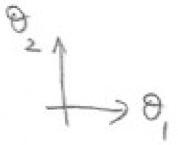
SLB 1 June 2007

Weak lensing shear power spectrum

(47),(48): $\gamma(\underline{\theta}) = \frac{1}{\pi}\int^{\infty} d^2\underline{\theta}' \; D(\underline{\theta} - \underline{\theta}')\, \kappa(\underline{\theta}')$

where $$D(\underline{\theta}) = \frac{\theta_2^2 - \theta_1^2 - 2i\,\theta_1\theta_2}{|\underline{\theta}|^4}$$

and $\underline{\theta} = (\theta_1, \theta_2)$



position on image plane.

Shear power spectrum can be defined in terms of the shear correlation functions

$$C_\gamma(\underline{\theta}) \equiv \langle \gamma(\underline{\theta}')\, \gamma^*(\underline{\theta}' + \underline{\theta}) \rangle_{\text{realisations of the Universe}} \qquad (60)$$

$$= \langle \gamma(\underline{\theta}')\, \gamma^*(\underline{\theta}' + \underline{\theta}) \rangle_{\underline{\theta}'}$$

i.e. measure the shear at positions $\underline{\theta}'$ and $\underline{\theta}' + \underline{\theta}$ i.e. find two galaxies of separation $\underline{\theta}$ and average over pairs of separation $\underline{\theta}$

By isotropy $C_\gamma(\underline{\theta}) = C_\gamma(\theta)$

$$\Rightarrow C_\gamma(\theta) = \left\langle \left(\gamma_1(\underline{\theta}') + i\,\gamma_2(\underline{\theta}')\right)\left(\gamma_1(\underline{\theta}'+\underline{\theta}) - i\,\gamma_2(\underline{\theta}'+\underline{\theta})\right)\right\rangle_{\underline{\theta}'}$$

$$= \langle \gamma_1(\underline{\theta}')\,\gamma_1(\underline{\theta}'+\underline{\theta})\rangle + \langle \gamma_2(\underline{\theta}')\,\gamma_2(\underline{\theta}'+\underline{\theta})\rangle_{\underline{\theta}'}$$

since $\langle \gamma_1(\underline{\theta}')\,\gamma_2(\underline{\theta}'+\underline{\theta})\rangle = 0$ since:

is equally likely as:

by parity

similarly $\langle \gamma_1(\underline{\theta}'+\underline{\theta})\,\gamma_2(\underline{\theta}')\rangle = 0$

$\Rightarrow$ Could write

$$C_\gamma(\theta) = C_{\gamma 1}(\theta) + C_{\gamma 2}(\theta) \qquad (61)$$

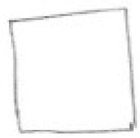
Define the shear power spectrum as the FT of the correlation function:

$$P_\gamma(\ell) \equiv \int d^2\underline{\theta}\; e^{-i\underline{\theta}.\underline{\ell}}\; C_\gamma(\theta) \qquad (62)$$

Work in Fourier space for simplicity (convolutions $\rightarrow$ multiplications)

Define the Fourier Transform in 2d:

$$\tilde{a}(\underline{\ell}) \equiv \int_{\Omega_s} d^2\underline{\theta}\; a(\underline{\theta})\; e^{-i\underline{\ell}.\underline{\theta}} \qquad (63)$$



area Ω_s

$\Rightarrow$ $$\tilde{\gamma}(\underline{\ell}) = \int_{\Omega_s} d^2\underline{\theta}\;\; \gamma(\underline{\theta})\; e^{-i\underline{\ell}.\underline{\theta}}$$

Where could write $\gamma(\underline{\theta}) = \frac{1}{\pi} D(\underline{\theta}) * K(\underline{\theta})$

$\Rightarrow$ use Wiener-Kinchin theorem $\rightarrow$

$$\tilde{\gamma}(\underline{\ell}) = \frac{1}{\pi}\; \tilde{D}(\underline{\ell})\, \tilde{K}(\underline{\ell})$$

$$\tilde{D}(\underline{\ell}) = \int_{\Omega_s} d^2\underline{\theta}\;\; D(\underline{\theta})\; e^{-i\underline{\ell}.\underline{\theta}}$$

$$= \int \frac{\theta_2^2}{|\underline{\theta}|^4}\, e^{-i\underline{\ell}.\underline{\theta}}\, d^2\underline{\theta} \;-\; \int \frac{\theta_1^2}{|\underline{\theta}|^4}\, e^{-i\underline{\ell}.\underline{\theta}}\, d^2\underline{\theta} \;-2i \int \frac{\theta_1\theta_2}{|\underline{\theta}|^4}\, e^{-i\underline{\ell}.\underline{\theta}}$$

e.g. see Bridle, Hobson, Lasenby + Saunders eqns 14-17

$$\ldots\ldots = \pi\left(\frac{\ell_1^2 - \ell_2^2 + 2i\,\ell_1\ell_2}{|\underline{\ell}|^2}\right)$$

Consider

$$\langle \tilde{\gamma}(\underline{\ell})\,\tilde{\gamma}^*(\underline{\ell}) \rangle_{\text{realisations}} = \frac{1}{\pi^2}\; \tilde{D}(\underline{\ell})\,\tilde{D}^*(\underline{\ell})\;\; \langle \tilde{K}(\underline{\ell})\,\tilde{K}^*(\underline{\ell}) \rangle$$

$$\tilde{D}(\underline{\ell})\,\tilde{D}^*(\underline{\ell}) = \frac{\pi^2}{|\underline{\ell}|^4}\;\; \left((\ell_1^2 - \ell_2^2) + 2i\ell_1\ell_2\right)\left((\ell_1^2 - \ell_2^2) - 2i\ell_1\ell_2\right)$$

$$= \frac{\pi^2}{|\underline{\ell}|^4}\left[(\ell_1^2-\ell_2^2)^2 + 4\ell_1^2\ell_2^2\right] = \frac{\pi^2}{(\ell_1^2+\ell_2^2)^2}\,(\ell_1^2+\ell_2^2)^2 = \pi^2$$

$\Rightarrow \quad \langle \tilde{\gamma}(\underline{\ell}) \, \tilde{\gamma}^*(\underline{\ell}) \rangle = \langle \tilde{\kappa}(\underline{\ell}) \, \tilde{\kappa}^*(\underline{\ell}) \rangle$

Now need to relate $\langle \tilde{\gamma}(\underline{\ell}) \tilde{\gamma}^*(\underline{\ell}) \rangle$ to $P_\gamma(\ell)$ ...

consider

$$\langle \tilde{\gamma}_i(\underline{\ell}) \, \tilde{\gamma}_i^*(\underline{\ell}') \rangle = \langle \int d^2\underline{\theta} \, \gamma_i(\underline{\theta}) \, e^{-i\underline{\ell}\cdot\underline{\theta}} \int d^2\underline{\theta}' \, \gamma_i(\underline{\theta}') \, e^{i\underline{\ell}'\cdot\underline{\theta}'} \rangle$$

$$= \int d^2\underline{\theta} \int d^2\underline{\theta}' \, \langle \gamma_i(\underline{\theta}) \, \gamma_i(\underline{\theta}') \rangle \, e^{-i\underline{\ell}\cdot\underline{\theta}} \, e^{i\underline{\ell}'\cdot\underline{\theta}'}$$

set $\underline{\theta}' = \underline{\theta} + \underline{\theta}''$

$$= \int d^2\underline{\theta} \int d^2\underline{\theta}'' \, \langle \gamma_i(\underline{\theta}) \, \gamma_i(\underline{\theta} + \underline{\theta}'') \rangle \, e^{-i\underline{\ell}\cdot\underline{\theta}} \, e^{i\underline{\ell}'\cdot\underline{\theta}} \, e^{i\underline{\ell}'\cdot\underline{\theta}''}$$

$$= \int d^2\underline{\theta}'' \, C_{\gamma i}(\underline{\theta}'') \, e^{i\underline{\ell}'\cdot\underline{\theta}''} \int d^2\underline{\theta} \, e^{i(\underline{\ell}' - \underline{\ell})\cdot\underline{\theta}}$$

for our FT defn we have

$$\delta_D(\underline{\ell}' - \underline{\ell}) = \frac{1}{(2\pi)^2} \int d^2\underline{\theta} \, e^{i(\underline{\ell}' - \underline{\ell})\cdot\underline{\theta}}$$

$$= (2\pi)^2 \int d^2\underline{\theta}'' \, C_{\gamma i}(\underline{\theta}'') \, e^{i\underline{\ell}'\cdot\underline{\theta}''} \, \delta_D(\underline{\ell}' - \underline{\ell})$$

$$= (2\pi)^2 \int d^2\underline{\theta} \, C_{\gamma i}(\theta) \, e^{-i\underline{\ell}\cdot\underline{\theta}} \, \delta_D(\underline{\ell}' - \underline{\ell})$$

because $C(\underline{\theta}) = C(-\underline{\theta})$ and $\underline{\theta}''$ is a dummy variable $\rightarrow \underline{\theta}$

$$\underline{b2} \qquad = (2\pi)^2 \, P_{\gamma i}(\ell) \, \delta_D(\underline{\ell}' - \underline{\ell})$$

Similar derivation for K ; $P_\gamma = P_{\gamma 1} + P_{\gamma 2}$; $\langle \tilde{\gamma}(\ell) \tilde{\gamma}(\ell') \rangle = \langle \tilde{\gamma}_1(\ell \ldots \rangle + \langle \tilde{\gamma}_2 \ldots \rangle$

$$\Rightarrow \quad \underline{\underline{P_\gamma(\ell) = P_K(\ell)}}$$