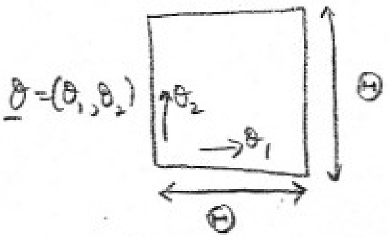
Derive the ~~lensing~~ convergence power spectrum

Assume we observe a finite area of flat sky




$\Omega_s = \Theta^2$

Consider contributions to lensing power spectrum from sheets at distances $\chi_i$ of width $\Delta\chi$, following Kaiser 1992 ApJ 388 Appendix A

Calculate lensing power spectrum for each sheet and assume sheets are thick enough to be statistically independent so can add up power spectra from each.

$$\kappa_i(\underline{\theta}) = q(\chi_i)\int_{\chi_i-\Delta\chi/2}^{\chi_i+\Delta\chi/2} d\chi\ \delta(\underline{x};\chi)$$

where $\underline{x} = \{\chi\theta_1, \chi\theta_2, \chi\}$

and $q(\chi) = \frac{3}{2}\frac{\Omega_m H_0^2}{c^2}\chi\int d\chi_s \frac{\chi_{ds}}{\chi_s} n(\chi_s)\ (1+z(\chi))$

Taking the FT of $\delta$:

$$\delta(\underline{x}) = \frac{1}{V}\sum_{\underline{k}_i}\tilde{\delta}(\underline{k}_i)\, e^{i\underline{k}_i\cdot\underline{x}} \quad \Rightarrow \delta(\underline{k}) = \int d^3\underline{x}\ \delta(\underline{x})\, e^{-i\underline{k}\cdot\underline{x}}$$

since $\int_V d^3\underline{x}\ e^{i(\underline{k}_i-\underline{k})\cdot\underline{x}} = V\delta(\underline{k}_i-\underline{k})$

$$\Rightarrow \tilde{\kappa}_i(\underline{\ell}) \equiv \int_{\Omega_s} d^2\underline{\theta}\ \kappa_i(\underline{\theta})\, e^{-i\underline{\ell}\cdot\underline{\theta}}$$

$$= q(\chi_i)\frac{1}{V}\sum_{\underline{k}_j}\tilde{\delta}(\underline{k}_j)\int_{\Omega_s} d^2\underline{\theta}\ e^{i(\underline{k}_j\cdot\underline{x}-\underline{\ell}\cdot\underline{\theta})}\int_{\chi_i-\Delta\chi/2}^{\chi_i+\Delta\chi/2} d\chi$$

$$\left[\begin{array}{l}\underline{k}\cdot\underline{x} = k_x\theta_1\chi + k_y\theta_2\chi + k_z\chi = \underline{k}_\perp\cdot\underline{\theta}\chi + k_z\chi \\ \Rightarrow \int_{\Omega_s} d^2\underline{\theta}\ e^{i(\underline{k}_\perp\chi-\underline{\ell})\cdot\underline{\theta}} = \Omega_s\,\delta(\underline{k}_\perp\chi-\underline{\ell})\end{array}\right.$$

$$= q(\chi_i)\frac{\Omega_s}{V}\sum_{k_{jz}}\sum_{k_\perp\chi}\tilde{\delta}(\underline{k}_j)\int_{\chi_i-\Delta\chi/2}^{\chi_i+\Delta\chi/2} d\chi\ e^{ik_{jz}\chi}\ \delta(\underline{k}_\perp\chi-\underline{\ell})$$

[where now $\underline{k}_j = (\ell_1/\chi, \ell_2/\chi, k_{jz})$

$$= q(\chi_i)\frac{\Omega_s}{V}\sum_{k_{jz}}\tilde{\delta}(\underline{k}_j)\ \Delta\chi\ j_0\!\left(\frac{k_{jz}\Delta\chi}{2}\right)$$

Derive the lensing power spectrum and its variance    Sun 10 Dec 2006 (2)

use $$\langle \tilde{\kappa}(\underline{\ell}) \tilde{\kappa}^*(\underline{\ell}') \rangle = \int_{\Omega_s} d^2\underline{\theta} \int_{\Omega_s} d^2\underline{\theta}' \langle \kappa(\underline{\theta}) \kappa(\underline{\theta}') \rangle e^{i(\underline{\ell}\cdot\underline{\theta} - \underline{\ell}'\cdot\underline{\theta}')}$$

$$= \Omega_s P_\kappa(\ell) \delta(\underline{\ell} - \underline{\ell}')$$

True for isotropic homogeneous random fields (not necessarily Gaussian) see Kaiser 1992 eqn A6

since $$P_\kappa(\ell) = \int_{\Omega_s} d^2\underline{\theta}\, \xi_\kappa(\theta) \frac{\sin(\ell\theta)}{\ell\theta}$$

and $$\xi_\kappa(\theta) = \langle \kappa(\underline{\theta}') \kappa(\underline{\theta}' + \underline{\theta}) \rangle$$

[ consider $\xi_\kappa = \delta(\theta) \Rightarrow P_\kappa(\ell) = 1$

as a way of getting the right normalisation

$$\Rightarrow \langle \tilde{\kappa}(\underline{\ell}) \tilde{\kappa}^*(\underline{\ell}') \rangle = \int_{\Omega_s} d^2\underline{\theta} \int_{\Omega_s} d^2\underline{\theta}' \, \delta(\underline{\theta}' - \underline{\theta}) \, e^{i(\underline{\ell}\cdot\underline{\theta} - \underline{\ell}'\cdot\underline{\theta}')}$$

$$= \int_{\Omega_s} d^2\underline{\theta} \, e^{i\underline{\theta}(\underline{\ell} - \underline{\ell}')}$$

$$= \Omega_s \delta(\underline{\ell} - \underline{\ell}')$$

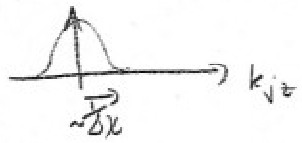
$$\Rightarrow \langle \tilde{\kappa}(\underline{\ell}) \tilde{\kappa}^*(\underline{\ell}') \rangle = \Omega_s P_\kappa(\ell) \delta(\underline{\ell} - \underline{\ell}')$$

similarly $$\langle \tilde{\delta}(\underline{k}) \tilde{\delta}^*(\underline{k}') \rangle = V P(k) \delta(\underline{k} - \underline{k}') = V P(k) \delta(k_z - k_z') \delta(\underline{\ell} - \underline{\ell}')$$

$$\Rightarrow \langle \tilde{\kappa}_i(\underline{\ell}) \tilde{\kappa}_i^*(\underline{\ell}') \rangle = q^2(\chi_i) \left(\frac{\Omega_s}{V}\right)^2 \sum_{k_{jz}} \sum_{k_{mz}} \langle \tilde{\delta}(\underline{k}_j) \tilde{\delta}^*(\underline{k}_m) \rangle \Delta\chi^2 \, j_0\left(\frac{k_{jz}\Delta\chi}{2}\right) j_0\left(\frac{k_{mz}\Delta\chi}{2}\right)$$

$$= q^2(\chi_i) \left(\frac{\Omega_s^2}{V}\right) \sum_{k_{jz}} P(k) \delta(\underline{\ell} - \underline{\ell}') \Delta\chi^2 \, j_0^2\left(\frac{k_{jz}\Delta\chi}{2}\right)$$

[ But width of Bessel fn is $\sim \frac{1}{\Delta\chi}$ in $k_{jz}$ space



$k = \sqrt{\frac{\ell^2}{\chi^2} + k_z^2}$ so since $\frac{\ell}{\chi} \gg \frac{1}{\Delta\chi}$ for large enough $\ell$

$\Rightarrow$ fourier modes ~ in plane of sky are the only ones that contribute

$\Rightarrow$ $k_{jz} \sim 0$, $\sum_{k_{jz}} j_0^2\left(\frac{k_{jz}\Delta\chi}{2}\right) \sim 1$

$$= q^2(\chi_i) \frac{\Omega_s^2}{V} \Delta\chi^2 \, P(\ell/\chi_i; \chi_i) \, \delta(\underline{\ell} - \underline{\ell}')$$

[ $V = \chi^2 \Omega_s \Delta\chi$

$$= q^2(\chi_i) \frac{\Omega_s \Delta\chi}{\chi_i^2} P(\ell/\chi_i; \chi_i) \, \delta(\underline{\ell} - \underline{\ell}')$$

Derive the lensing power spectrum and its variance    Sun 10 Dec 2006 (3)

→ For this slice

$$P_{\kappa i}(l) = q^2(\chi_i) \frac{\Delta\chi}{\chi_i^2} P(l/\chi_i; \chi_i)$$

→ Adding up over slices

$$P_\kappa(l) = \int \frac{d\chi}{\chi} q^2(\chi) P(l/\chi; \chi) \qquad |||$$