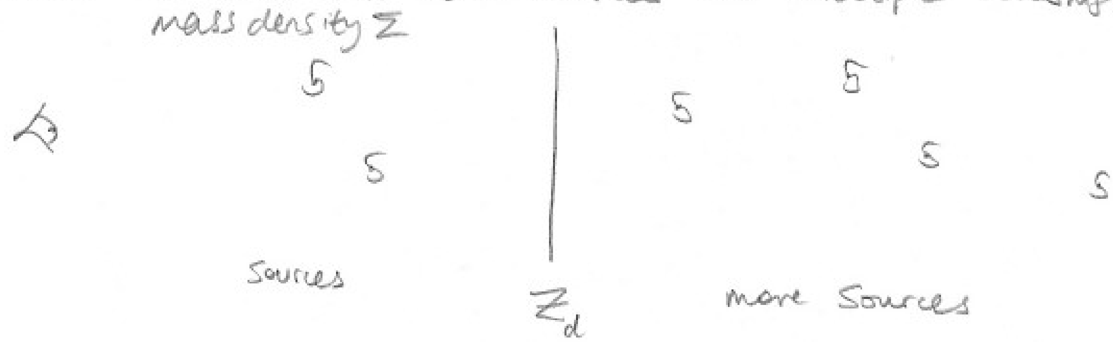
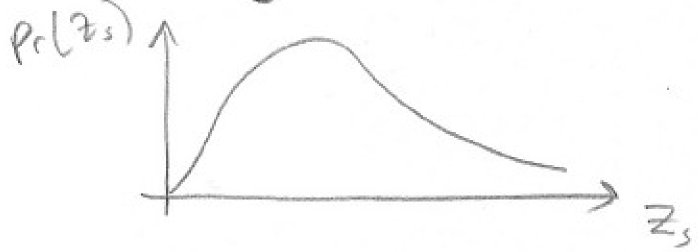
SLB 1 June 2007

The convergence for multiple source planes

Consider a thin lens and sources at multiple redshifts.



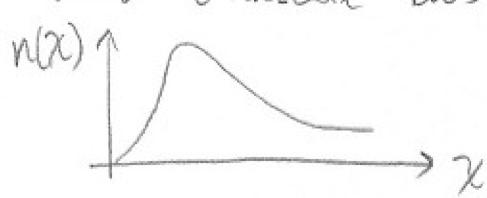
The sources are distributed in redshift according to a probability distribution



where $\int P_r(z_s)\,dz_s = 1$

Where the fraction of galaxies with redshifts between z_s and $z_s + \Delta z_s$ is $P_r(z_s)\,\Delta z_s$

[or could translate this into comoving distance χ instead:



fraction of gals in χ to $\Delta\chi$ is $n(\chi)\Delta\chi$

note $n(\chi) \neq P_r(z_s)$ for $z_s = z(\chi)$

but $P_r(z_s)\Delta z_s = n(\chi)\Delta\chi$ for $z_s = z(\chi)$, $z_s + \Delta z_s = z(\chi + \Delta\chi)$

$$\rightarrow \quad n(\chi) = P_r(z_s)\frac{dz}{d\chi}$$
]

for most purposes we can consider an effective $K \equiv \dfrac{\Sigma}{\Sigma_{crit}}$ for this population

$$K_{eff} = \int_{z_d}^{\infty} K_{source\ i}\; P_r(z_s)\,dz_s$$

z_d ← because sources between us and lens are not lensed → do not contribute.

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$$\Rightarrow \quad \kappa_{eff} = \int_{z_d}^{\infty} \frac{\Sigma}{\Sigma_{crit}} \, Pr(z_s) \, dz_s$$

(41) defn of Σ_{crit}

$$= \Sigma \int_{z_d}^{\infty} \frac{4\pi G}{c^2} \frac{D_d D_{ds}}{D_s} \, Pr(z_s) \, dz_s$$

mass density

$$= \Sigma \, \frac{4\pi G}{c^2} D_d \int_{\chi_d}^{\infty} d\chi_s \, \frac{D_{ds}}{D_s} \, n(\chi_s)$$