Axially symmetric lenses

$\theta \equiv |\underline{\theta}|$

see Schneider eqn 40

Generalise (10), (21), (22) for axially symmetric lens. $\Rightarrow K(\underline{\theta}) = K(\theta)$ (49)

Use Birkhoff's theorem: mass enclosed by sphere of radius r acts gravitationally like a point at the centre, of mass = enclosed mass (for spherically symmetric mass dist^ns only).

Also, lens bend angle does not care exactly how mass is distributed along line of sight if all roughly at D_d.

$\Rightarrow$ in eqn (1) this can be mass enclosed at radius ξ

follow this through derivation of eqn (10) $\Rightarrow$

(10) becomes

$$\underline{\alpha} = \frac{D_{ds}}{D_s D_d} \frac{4G}{c^2} \frac{\underline{\theta}}{|\underline{\theta}|^2} M_{encl} \quad (50)$$

plus use (5):

$$\alpha = \frac{D_{ds}}{D_s D_d} \frac{4G}{c^2} \frac{1}{|\underline{\theta}|} M_{encl} \quad (51)$$

Using notation in (36)

$$\Rightarrow M_{encl}(\underline{\theta}) \equiv M_{encl}(\theta) = \int_{|\underline{\theta}'|=0}^{|\underline{\theta}'|<|\underline{\theta}|} dm = \int \Sigma(D_d \underline{\theta}) D_d^2 \, d^2\underline{\theta}$$

$$= \Sigma_{crit} D_d^2 \int_{\theta'=0}^{\theta} K(\underline{\theta}') \underbrace{2\pi \theta' d\theta'}_{d^2\theta' \text{ becomes this}} \quad (52)$$

since now lets integrate in annuli

$$\Rightarrow \alpha = \frac{2}{|\underline{\theta}|} \int_{\theta'=0}^{\theta} K(\theta') \, \theta' d\theta'$$

(e.g. Schneider eqn 34) (53)

use Σ_{crit} def